

# The Gauge Dual of Gauged $\mathcal{N} = 8$ Supergravity Theory

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## Abstract

The most general  $SU(3)$ -singlet space of gauged  $\mathcal{N} = 8$  supergravity in four-dimensions is studied recently. The  $SU(3)$ -invariant six scalar fields are realized by six real four-forms. A family of holographic  $\mathcal{N} = 1$  supersymmetric RG flows on M2-branes in three-dimensions is described. This family of flows is driven by three independent mass parameters from the  $\mathcal{N} = 8$   $SO(8)$  theory and is controlled by two IR fixed points,  $\mathcal{N} = 1$   $G_2$ -invariant one and  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant one. The generic flow with arbitrary mass parameters is  $\mathcal{N} = 1$  supersymmetric and reaches to the  $\mathcal{N} = 2$   $SU(3) \times U(1)$  fixed point where the three masses become identical. A particular  $\mathcal{N} = 1$  supersymmetric  $SU(3)$ -preserving RG flow from the  $\mathcal{N} = 1$   $G_2$ -invariant fixed point to the  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point is also discussed.

# 1 Introduction

The three-dimensional  $\mathcal{N} = 6$   $U(N) \times U(N)$  Chern-Simons matter theories with level  $k$  can be regarded as the low energy limit of  $N$  M2-branes at  $\mathbf{C}^4/\mathbf{Z}_k$  singularity [1]. The coupling of this theory may be thought of as  $\frac{1}{k}$  and so this is weakly coupled for large  $k$ . For  $k = 1, 2$ , the full  $\mathcal{N} = 8$  supersymmetry is preserved with  $SO(8)$  R-symmetry and this becomes strongly coupled theory. For  $k > 2$ , the supersymmetry is broken to  $\mathcal{N} = 6$  and R-symmetry is broken to  $SO(6)$ .

The renormalization group(RG) flow between the ultraviolet(UV) fixed point and the infrared(IR) fixed point of the three-dimensional field theory can be described from gauged  $\mathcal{N} = 8$  supergravity theory in four-dimensions via AdS/CFT correspondence [2, 3, 4]. The holographic supersymmetric RG flow equations connecting  $\mathcal{N} = 8$   $SO(8)$ -invariant fixed point to  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point have been found in [5, 6](See also [7] for earlier work). The other holographic supersymmetric RG flow equations from  $\mathcal{N} = 8$   $SO(8)$ -invariant fixed point to  $\mathcal{N} = 1$   $G_2$ -invariant fixed point also have been studied in [6, 8, 9](See also [10, 11] for previous work on the critical point and the metric respectively). The exact solutions to the  $M$ -theory lift of these supersymmetric RG flows have been constructed in [12, 8] respectively. There exist three supersymmetric critical points in gauged  $\mathcal{N} = 8$  supergravity theory.

The mass deformed  $U(2) \times U(2)$  Chern-Simons matter theory with level  $k = 1$  or  $k = 2$  preserving global  $SU(3) \times U(1)$  symmetry has been studied in [13, 14, 15, 16] by adding a single mass term for  $\mathcal{N} = 2$  superfield. The mass deformation for this theory preserving  $G_2$  symmetry has been described in [17] by adding a single mass term for  $\mathcal{N} = 1$  superfield. The nonsupersymmetric RG flow equations preserving  $SO(7)^\pm$  symmetry have been discussed in [18] by looking at the previous work [6] closely. The holographic RG flow equations connecting  $\mathcal{N} = 1$   $G_2$ -invariant fixed point to  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point have been found in [19] by computing and analyzing the mass eigenvalues in gauged  $\mathcal{N} = 8$  supergravity. This is the last flow connecting the remaining two nontrivial supersymmetric critical points. Moreover, the  $\mathcal{N} = 4$  and  $\mathcal{N} = 8$  supersymmetric RG flows have been studied in [20] by adding the appropriate mass terms.

The gauged  $\mathcal{N} = 8$  supergravity in four-dimensions has a scalar potential which is a function of 70 scalars, in general [21]. For one possible embedding of  $SU(3)$ , two 35-dimensional representations of  $SO(8)$  contain three singlets. Then the set of 70 scalars in gauged  $\mathcal{N} = 8$  supergravity contains six singlets of  $SU(3)$ . It is known [10] that  $SU(3)$ -singlet space with a breaking of the  $SO(8)$  gauge group into a group which contains  $SU(3)$  may be written in terms of the action of  $SU(2) \times U(1)$  subgroup of  $SU(8)$  on 70-dimensional representation in

the space of self-dual complex four-forms together with two real parameters. Instead of taking the  $SU(2)$  group as a subgroup, one can have any  $2 \times 2$  unitary matrix  $U(2)$ . Then this  $U(2)$  group element is realized by four real parameters and moreover there are two additional real parameters. Recently, a new scalar potential for  $U(2) \times U(1)$  subgroup of  $SU(8)$  of gauged  $\mathcal{N} = 8$  supergravity has been found in [22]. Although  $A_1$  tensor of the theory depends on the parameters on  $U(2)$  group, after diagonalizing the  $A_1$  tensor, the two eigenvalues become simple and they can be obtained from those eigenvalues of [6] by field redefinitions. This new scalar potential has alternative form that can be read off from the scalar potential in [10]. The nontrivial BPS domain-wall solutions for restricted scalar submanifold from direct extremization of energy-density is also discussed.

In this paper, we would like to describe the corresponding Chern-Simons matter theory in three-dimensions dual to turning on the six scalar fields for  $SU(3)$ -singlet space of gauged  $\mathcal{N} = 8$  supergravity. The phase structure of the flows with two mass parameters was studied in [19] through the analysis of gravity dual that corresponds to turn on the four scalar fields for the  $SU(3)$ -singlet space. The family of our flows is driven by three independent mass parameters from the  $\mathcal{N} = 8$   $SO(8)$  theory and is controlled by two IR fixed points,  $\mathcal{N} = 1$   $G_2$ -invariant fixed point and  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point. We would like to see how the extra mass parameter changes the flows connecting these three supersymmetric fixed points. This can be seen from the supergravity dual analysis where the extra two supergravity fields become the other two fields respectively. The generic  $\mathcal{N} = 1$  supersymmetric flow with arbitrary mass parameters approaches to the  $\mathcal{N} = 2$   $SU(3) \times U(1)$  fixed point where the three masses become identical. A particular  $\mathcal{N} = 1$  supersymmetric  $SU(3)$ -invariant RG flow from the  $\mathcal{N} = 1$   $G_2$ -invariant fixed point to the  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point occurs.

In section 2, we review the construction of scalar potential given in [22] for  $U(2) \times U(1)$  subgroup of  $SU(8)$  of gauged  $\mathcal{N} = 8$  supergravity. We focus on the nontrivial supersymmetric critical points and describe the BPS domain-wall solutions for restricted scalar submanifold from direct extremization of energy-density. We also consider the supersymmetric flows around three supersymmetric critical points.

In section 3, we deform the BL theory by generalizing the possible mass terms preserving the common  $SU(3)$  symmetry. We concentrate on  $\mathcal{N} = 1$  and  $\mathcal{N} = 2$  supersymmetries. We obtain the deformed superpotential with three mass parameters. Based on the supergravity dual theory found in section 2, we analyze the features of RG flows in these parameter space by looking at the role of the extra mass parameter.

In section 4, we summarize what we have obtained in this paper and make some comments on the future directions.

## 2 The holographic supersymmetric RG flows in four dimensions

The gauged  $\mathcal{N} = 8$  supergravity theory contains self-interaction of a single massless  $\mathcal{N} = 8$  supermultiplet with local  $SO(8)$ - and local  $SU(8)$ -invariances. The 70-real, physical scalars of gauged  $\mathcal{N} = 8$  supergravity parametrize the coset space  $E_{7(7)}/SU(8)$ . The most general  $SU(3)$ -singlet space parametrized by six fields (denoted by  $\lambda, \lambda', \rho, \alpha, \phi, \varphi$ ) among 70-fields is represented by the 56-bein of the fundamental representation of  $E_{7(7)}$ . After exponentiating the  $SU(3)$ -singlet which is  $56 \times 56$  matrix, this 56-bein can be decomposed into two independent 28-beins. From the T-tensor defined by these 28-beins, the new scalar potential [23, 24] parametrized by the above six fields of the theory is found in [22] via long, tedious computations. Although the scalar potential looks very complicated at first sight, it can be rewritten as the scalar potential [10] parametrized by four fields through simple field redefinitions, according to the observation of [22]. This feature makes easier to analyze the critical points of the scalar potential.

As one of the angular variables is equal to the other angular variable ( $\varphi = \phi$ ), then the scalar potential with five independent fields has a sum of the square of superpotential and the squares of derivatives of superpotential with respect to the fields. More explicitly the scalar potential depends on  $\lambda, \sqrt{\lambda'^2 + \rho^2}, \alpha$  and  $\phi$  and the dependence of the extra field  $\rho$  occurs only in the form of  $\sqrt{\lambda'^2 + \rho^2}$ . That is, it depends on four independent quantities. From the kinetic terms of the theory, further constraint on the field, i.e., the condition that two fields are equal ( $\rho = \lambda'$ ), gives a simple relation between the scalar potential and the superpotential because the expression  $\sqrt{\lambda'^2 + \rho^2}$  becomes  $\sqrt{2}\lambda'$  in this case. This is required to possess the BPS bound of the energy-density. Therefore, the extra fields  $(\rho, \varphi)$  we turn on newly in the four-forms [22] should be the same as  $(\lambda', \phi)$  respectively along the whole RG flows we are considering. We'll use this property when we discuss the IR behavior of the family of flows in terms of deformed mass terms in the superpotential in next section. Finally, the scalar potential reduces to the one [10] with a simple rescale  $\sqrt{2}$  on the field  $\lambda'$ .

In summary, the reduced supergravity potential on the most general  $SU(3)$ -invariant sector, by putting the constraints  $\varphi = \phi$  and  $\rho = \lambda'$ , is then given by [22]

$$V(\lambda, \lambda', \rho; \alpha, \phi, \varphi)|_{\varphi=\phi, \rho=\lambda'} = g^2 \left[ \frac{16}{3} \left| \frac{\partial z_3}{\partial \lambda} \right|^2 + 2 \left| \frac{\partial z_3}{\partial \lambda'} \right|^2 - 6 |z_3|^2 \right], \quad (2.1)$$

where the complex superpotential  $z_3$  in (2.1), which is an eigenvalue of  $A_1$  tensor of the theory,

has the following form [22]

$$z_3(\lambda, \lambda', \rho; \alpha, \phi, \varphi) = 6e^{i(\alpha+2\beta)}p^2qr^2t^2 + 6e^{2i(\alpha+\beta)}pq^2r^2t^2 + p^3(r^4 + e^{4i\beta}t^4) + e^{3i\alpha}q^3(r^4 + e^{4i\beta}t^4), \quad (2.2)$$

and the hyperbolic functions  $p, q, r$  and  $t$  that depend on  $\lambda$  or  $\lambda'$  and trigonometric function  $\beta$  are introduced and they are reduced to as follows after imposing the conditions  $\varphi = \phi, \rho = \lambda'$ :

$$\begin{aligned} p &\equiv \cosh\left(\frac{\lambda}{2\sqrt{2}}\right), & q &\equiv \sinh\left(\frac{\lambda}{2\sqrt{2}}\right), \\ r &\equiv \cosh\left(\frac{\sqrt{\lambda'^2 + \rho^2}}{2\sqrt{2}}\right)|_{\rho=\lambda'} = \cosh\left(\frac{\lambda'}{2}\right), & t &\equiv \sinh\left(\frac{\sqrt{\lambda'^2 + \rho^2}}{2\sqrt{2}}\right)|_{\rho=\lambda'} = \sinh\left(\frac{\lambda'}{2}\right), \\ \beta &\equiv \frac{1}{2} \cos^{-1}\left(\frac{\lambda'^2 \cos 2\phi + \rho^2 \cos 2\varphi}{\lambda'^2 + \rho^2}\right)|_{\varphi=\phi, \rho=\lambda'} = \phi. \end{aligned} \quad (2.3)$$

There exist six critical points of this scalar potential. Three of them are supersymmetric while the other three are nonsupersymmetric. The symmetry group has a common  $SU(3)$  group. Let us present the three supersymmetric ones.

- $SO(8)$  critical point

This occurs at  $\lambda = 0 = \lambda' = \rho$ , the cosmological constant is  $\Lambda = -6g^2$  (and  $W = 1$ ) and the  $\mathcal{N} = 8$  supersymmetry is preserved.

- $G_2$  critical point

There is a critical point at  $\lambda = \sqrt{2} \sinh^{-1}(\sqrt{\frac{2}{5}}(\sqrt{3}-1)) = \frac{\lambda'}{\sqrt{2}} = \frac{\rho}{\sqrt{2}}$  as well as  $\alpha = \cos^{-1}(\frac{1}{2}\sqrt{3-\sqrt{3}}) = \phi = \varphi$  and the cosmological constant is  $\Lambda = -\frac{216\sqrt{2}}{25\sqrt{5}} \cdot 3^{\frac{1}{4}}g^2$  (and  $W = \sqrt{\frac{36\sqrt{2}\cdot 3^{\frac{1}{4}}}{25\sqrt{5}}}$ ). This has an unbroken  $\mathcal{N} = 1$  supersymmetry.

- $SU(3) \times U(1)$  critical point

Finally, there is a critical point at  $\lambda = \sqrt{2} \sinh^{-1}(\frac{1}{\sqrt{3}})$ ,  $\frac{\lambda'}{\sqrt{2}} = \sqrt{2} \sinh^{-1}(\frac{1}{\sqrt{2}}) = \frac{\rho}{\sqrt{2}}$ ,  $\alpha = 0$  and  $\phi = \frac{\pi}{2} = \varphi$  and the cosmological constant is  $\Lambda = -\frac{9\sqrt{3}}{2}g^2$  (and  $W = \frac{3^{\frac{3}{4}}}{2}$ ). This critical point has an unbroken  $\mathcal{N} = 2$  supersymmetry.

The supersymmetric flow equations together with (2.3) are given by [22]

$$\begin{aligned} \frac{d\lambda}{dr} &= \frac{8\sqrt{2}}{3}g\partial_\lambda W, & \frac{d\lambda'}{dr} &= \sqrt{2}g\partial_{\lambda'} W = \frac{d\rho}{dr}, \\ \frac{d\alpha}{dr} &= \frac{\sqrt{2}}{3p^2q^2}g\partial_\alpha W, & \frac{d\phi}{dr} &= \frac{\sqrt{2}}{4r^2t^2}g\partial_\phi W = \frac{d\varphi}{dr}, \\ \frac{dA}{dr} &= -\sqrt{2}gW, \end{aligned} \quad (2.4)$$

where the real superpotential is given by

$$W = |z_3|, \quad (2.5)$$

with (2.2) and (2.3). The scale factor  $A(r)$  in the last equation of (2.4) appears in the four-dimensional metric  $ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2$  with three-dimensional metric  $\eta_{\mu\nu} = (-, +, +)$ . The superpotential  $W$  appearing in (2.4) is the same as the one in [6] except the factor  $\sqrt{2}$  in front of  $\lambda'$  (or  $\rho$ ) for the hyperbolic functions of (2.3). Moreover, there exist extra two first-order differential equations on  $\rho$  and  $\varphi$  in (2.4). Let us emphasize that although we have started with six  $SU(3)$ -singlet fields rather than four  $SU(3)$ -singlet fields, it turns out the scalar potential (2.1) and the superpotential (2.5) reduce to those in four  $SU(3)$ -singlet fields up to rescale we mentioned before, with extra conditions  $\varphi = \phi$  and  $\rho = \lambda'$ . Then the critical points reduce to those for four  $SU(3)$ -singlet fields <sup>1</sup>.

In order to understand the flows around the three supersymmetric fixed points,  $\mathcal{N} = 8$   $SO(8)$ ,  $\mathcal{N} = 2$   $SU(3) \times U(1)$  and  $\mathcal{N} = 1$   $G_2$ , one can analyze the scaling dimensions of the operators and the linearization of the flow equations (2.4) can be done around these fixed points. From the mass spectrum formula on  $\mathbf{S}^7$ , the  $\mathbf{35}_c$  pseudo-scalars of  $SO(8)$  can be identified with the second derivative of the scalar potential evaluated at the  $SO(8)$  fixed point and correspond to the conformal primaries of  $\Delta = 2$  which consists of quadratic fermions in the irreducible representations  $\mathbf{8}_c$  of  $SO(8)$ . Here the  $SO(8)$  coupling constant  $g$  is given in terms of the radius of  $AdS_4$  via  $g = \frac{1}{\sqrt{2}r_{UV}}$ . The asymptotic behavior of  $A(r)$  is given by  $A(r) \rightarrow \frac{r}{r_{UV}} + \text{const}$  for  $r \rightarrow \infty$ . On the other hand, the  $\mathbf{35}_v$  scalars of  $SO(8)$  correspond to the other second derivative of the scalar potential evaluated at the  $SO(8)$  fixed point. The conformal dimension is  $\Delta = 1$  which consists of quadratic bosons in the irreducible representations  $\mathbf{8}_v$  of  $SO(8)$ . The holographic RG flow equations connecting  $\mathcal{N} = 8$   $SO(8)$ -invariant fixed point to  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point were found in [5] and the other holographic RG flow equations from  $\mathcal{N} = 8$   $SO(8)$ -invariant fixed point to  $\mathcal{N} = 1$   $G_2$ -invariant fixed point also were studied in [8].

Now the remaining holographic RG flow equations connect between  $\mathcal{N} = 1$   $G_2$ -invariant fixed point and  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point. Compared with the previous RG flow equations in previous paragraph where we have dealt with only two fields, these RG flow equations should keep all the four  $SU(3)$ -singlet fields. Recall that the scalar potential and the superpotential reduce to those in four  $SU(3)$ -singlet fields. For the flows near the neighborhood of other two supersymmetric fixed points,  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point and  $\mathcal{N} = 1$   $G_2$ -invariant fixed point, one can compute the mass eigenvalues at the critical points [19]. At the  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point, the eigenvalues are given by  $\frac{1 \pm \sqrt{17}}{2}$  and  $\frac{3 \pm \sqrt{17}}{2}$  which have two negative values corresponding to irrelevant operators flowing

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<sup>1</sup>The supergravity kink interpolates between an asymptotically  $AdS_4$  space in the  $UV(r \rightarrow \infty)$  and another in the  $IR(r \rightarrow -\infty)$  and this can be thought of as an explicit construction of a RG flow between a UV fixed point and an IR fixed point of the boundary field theory. For  $AdS_5$  compactification, see the earlier work [25].

into the IR fixed point and have two positive values corresponding to relevant operators or vacuum expectation values driving the flows away from the IR fixed point. At the  $\mathcal{N} = 1$   $G_2$ -invariant fixed point, the eigenvalues are  $1 \pm \sqrt{6}$  and  $1 \pm \frac{1}{\sqrt{6}}$  which have one negative value and which have three positive values. The two of positive values ( $1 \pm \frac{1}{\sqrt{6}}$ ), which are less than  $\frac{3}{2}$ , correspond to non-normalizable modes and should be interpreted as a perturbation of Lagrangian we are interested in. The sum of these eigenvalues is 2. These represent the fermionic and bosonic mass terms generating the  $\mathcal{N} = 1$  supersymmetric flow from the  $\mathcal{N} = 1$   $G_2$ -invariant fixed point to  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point. They claim that the supergravity predicts an anomalous dimension of  $\pm \frac{1}{\sqrt{6}}$  that drives this  $\mathcal{N} = 1$  flow [19].

### 3 The holographic supersymmetric M2-brane flows in three dimensions

Let us recall that the six four-forms consisting of three self-dual and three anti-self-dual are regarded as six fields in gauged  $\mathcal{N} = 8$  supergravity and they are given explicitly by [22, 19, 26]

$$\begin{aligned} F_1^\pm &= \varepsilon_\pm [(\delta_{IJKL}^{1234} \pm \delta_{IJKL}^{5678}) + (\delta_{IJKL}^{1256} \pm \delta_{IJKL}^{3478}) + (\delta_{IJKL}^{3456} \pm \delta_{IJKL}^{1278})], \\ F_2^\pm &= \varepsilon_\pm [-(\delta_{IJKL}^{1357} \pm \delta_{IJKL}^{2468}) + (\delta_{IJKL}^{2457} \pm \delta_{IJKL}^{1368}) + (\delta_{IJKL}^{2367} \pm \delta_{IJKL}^{1458}) + (\delta_{IJKL}^{1467} \pm \delta_{IJKL}^{2358})], \\ F_3^\pm &= \varepsilon_\pm [(\delta_{IJKL}^{2467} \mp \delta_{IJKL}^{1358}) - (\delta_{IJKL}^{1367} \mp \delta_{IJKL}^{2458}) - (\delta_{IJKL}^{1457} \mp \delta_{IJKL}^{2368}) - (\delta_{IJKL}^{2357} \mp \delta_{IJKL}^{1468})] \end{aligned} \quad (3.1)$$

where  $\varepsilon_+ = 1$  for scalars and  $\varepsilon_- = i$  for pseudoscalars. The indices  $I, J, K, L$  of these four-forms are the Cartesian coordinates in the vector representation of  $SO(8)$  of gauged  $\mathcal{N} = 8$  supergravity theory. In particular, the index 7 and index 8 in (3.1) which are stabilized by the subgroup  $SO(6)$  of  $SO(8)$  appear simultaneously in  $F_1^\pm$  (in the indices of 5678, 3478, and 1278) while they appear in  $F_2^\pm$  and  $F_3^\pm$  independently. The  $F_2^\pm$  are related to the  $F_3^\pm$  by replacing the index 7 with the index 8 and vice versa up to the signs. Note that all these four-forms are invariant under the  $SU(3)$  subgroup of  $SO(6) \subset SO(8)$ .

The mass deformation of BL theory [27, 28, 29] ( $U(2) \times U(2)$  Chern-Simons matter theory with level  $k = 1$  or  $k = 2$  is equivalent to the two M2-branes of BL theory) has the following fermion (32 component Majorana spinors of  $SO(1, 10)$  subject to a chiral condition on the M2-brane world volume which leads to 16 real degrees of freedom) mass terms (See also [30, 31, 32]) in the Lagrangian

$$\begin{aligned} \mathcal{L}_{f.m.} &= -\frac{i}{2} h_{ab} \bar{\Psi}^a (m_1 \Gamma^{78910} + m_2 \Gamma^{56910} + m_3 \Gamma^{5678} \\ &\quad - m_4 \Gamma^{46810} + m_5 \Gamma^{4679} + m_6 \Gamma^{4589} + m_7 \Gamma^{45710} \\ &\quad - m_9 \Gamma^{35710} - m_{10} \Gamma^{3589} - m_{11} \Gamma^{3679} + m_{12} \Gamma^{36810}) \Psi^b. \end{aligned} \quad (3.2)$$

Intentionally, we put minus signs in the four mass terms corresponding to the self-dual four-forms that have minus signs in (3.1) above. We denote only  $SO(4)$  gauge index  $a$  in the fermion and the spinor indices for fermions are omitted. The mass terms in (3.2) consist of fourth order product of the eleven-dimensional Gamma matrices  $\Gamma^\mu$  where  $\mu = 0, 1, 2, \dots, 10$  which satisfy the usual anticommutator relations with the metric. The spinors are eigenvectors of both  $\Gamma^{012}$  and  $\Gamma^{345678910}$ . The  $SO(8)$  vector indices of (3.1) can be seen from (3.2) by subtracting two from those in (3.2). Namely, the indices 78910 containing  $m_1$  in (3.2) correspond to the indices 5678 of the second term in  $F_1^\pm$  of (3.1). The spinor indices are contracted with those of Gamma matrices. The presence of  $F_3^\pm$  in the internal four-forms flux corresponding to the supergravity fields  $(\rho, \varphi)$  in this paper reflects the last line of (3.2) by choosing the indices 1358, 1367, 1457 and 1468 from (3.1).

By assuming the linearity in masses, the fermionic supersymmetry transformation gets modified by the following terms which are linear in the mass

$$\begin{aligned} \delta_{mass} \Psi^a &= (m_1 \Gamma^{78910} + m_2 \Gamma^{56910} + m_3 \Gamma^{5678} - m_4 \Gamma^{46810} + m_5 \Gamma^{4679} + m_6 \Gamma^{4589} + m_7 \Gamma^{45710} \\ &- m_9 \Gamma^{35710} - m_{10} \Gamma^{3589} - m_{11} \Gamma^{3679} + m_{12} \Gamma^{36810}) X_I^a \Gamma^I \epsilon. \end{aligned} \quad (3.3)$$

The supersymmetry parameter  $\epsilon$  in (3.3) is also eigenvectors of both  $\Gamma^{012}$  and  $\Gamma^{345678910}$ . Then the linear or quadratic terms in masses to the Lagrangian should be determined. Let us introduce the bosonic mass terms [30, 31, 32]

$$\mathcal{L}_{b.m.} = -\frac{1}{2} h_{ab} X_I^a (m^2)_{IJ} X_J^b, \quad (3.4)$$

and we would like to determine  $(m^2)_{IJ}$  in (3.4) explicitly which will play the role of mass-deformed superpotential together with (3.2) later. Of course, before the mass deformation, the superpotential of original theory contains the quartic terms in the  $\mathcal{N} = 1$  superfield having the bosonic field  $X_I^a$  as fermionic independent term.

From the supersymmetry transformation for the bosonic fields  $\delta X_I^a = i\bar{\epsilon} \Gamma_I \Psi^a$  and the supersymmetry variation for the spinors (3.3), one obtains the quadratic mass terms in the Lagrangian which contains the bosonic mass terms (3.4) and the fermionic mass terms (3.2). The variation is as follows

$$\begin{aligned} \delta \mathcal{L} &= i h_{ab} X_I^a (m^2)_{IJ} \bar{\Psi}^b \Gamma_J \epsilon - i h_{ab} \bar{\Psi}^a (m_1 \Gamma^{78910} + m_2 \Gamma^{56910} + m_3 \Gamma^{5678} \\ &- m_4 \Gamma^{46810} + m_5 \Gamma^{4679} + m_6 \Gamma^{4589} + m_7 \Gamma^{45710} \\ &- m_9 \Gamma^{35710} - m_{10} \Gamma^{3589} - m_{11} \Gamma^{3679} + m_{12} \Gamma^{36810})^2 X_I^b \Gamma^I \epsilon. \end{aligned} \quad (3.5)$$

Then the possible supersymmetric mass deformations in (3.5) are characterized by the bosonic mass terms  $(m^2)_{IJ}$ , the fourth order product of the Gamma matrices  $\sum_i m_i \Gamma_i^{(4)}$  with fermion



mass terms and the supersymmetry parameter  $\epsilon$ . In order to vanish the variation (3.5), the relation between the mass terms of boson and fermion with the supersymmetry parameter should be satisfied

$$(m^2)_{IJ}\Gamma^J\epsilon = (m_1\Gamma^{78910} + m_2\Gamma^{56910} + m_3\Gamma^{5678} - m_4\Gamma^{46810} + m_5\Gamma^{4679} + m_6\Gamma^{4589} + m_7\Gamma^{45710} - m_9\Gamma^{35710} - m_{10}\Gamma^{3589} - m_{11}\Gamma^{3679} + m_{12}\Gamma^{36810})^2 \Gamma_I\epsilon. \quad (3.6)$$

Let us compute the quadratic mass terms in the right hand side of (3.6) in order to determine the bosonic mass terms of the left hand side. The 38 terms among them are given by

$$\begin{aligned} & \sum_{i=1}^7 m_i^2 + \sum_{i=9}^{12} m_i^2 + 2 [(-m_1m_2 - m_4m_7 - m_5m_6 - m_9m_{12} - m_{10}m_{11})\Gamma^{5678} \\ & + (-m_1m_3 - m_4m_6 - m_5m_7 - m_9m_{11} - m_{10}m_{12})\Gamma^{56910} \\ & + (-m_1m_4 - m_2m_7 + m_3m_6)\Gamma^{4679} + (m_1m_5 + m_2m_6 + m_3m_7)\Gamma^{46810} \\ & + (-m_1m_6 - m_2m_5 - m_3m_4)\Gamma^{45710} + (-m_1m_7 - m_2m_4 - m_3m_5)\Gamma^{4589} \\ & + (-m_2m_3 - m_4m_5 - m_6m_7 - m_9m_{10} - m_{11}m_{12})\Gamma^{78910}] , \end{aligned} \quad (3.7)$$

and the remaining 28 terms are given by

$$\begin{aligned} & 2 [(m_1m_9 + m_2m_{12} + m_3m_{11})\Gamma^{3589} + (m_1m_{10} + m_2m_{11} + m_3m_{12})\Gamma^{35710} \\ & + (-m_1m_{11} - m_2m_{10} - m_3m_9)\Gamma^{36810} + (m_1m_{12} + m_2m_9 + m_3m_{10})\Gamma^{3679} \\ & + (-m_4m_9 - m_5m_{10} - m_6m_{11} - m_7m_{12})\Gamma^{345678} \\ & + (-m_4m_{10} - m_5m_9 - m_6m_{12} - m_7m_{11})\Gamma^{3456910} \\ & + (-m_4m_{11} - m_5m_{12} - m_6m_9 - m_7m_{10})\Gamma^{3478910} \\ & + (m_4m_{12} + m_5m_{11} + m_6m_{10} + m_7m_9)\Gamma^{34}] . \end{aligned} \quad (3.8)$$

So far, this holds for any supersymmetry parameter. From now on we need to classify the possible supersymmetric mass deformations by constraining the supersymmetry parameter.

Let us describe the possible mass deformations based on the number of supersymmetry.

- $\mathcal{N} = 1$  supersymmetry

The 1/8 BPS condition (the number of supersymmetry is 2) has the following constraints on the supersymmetry parameter  $\epsilon$  [17]

$$\Gamma^{5678}\epsilon = \Gamma^{56910}\epsilon = \Gamma^{78910}\epsilon = \Gamma^{46810}\epsilon = -\Gamma^{4679}\epsilon = -\Gamma^{4589}\epsilon = -\Gamma^{45710}\epsilon = -\epsilon. \quad (3.9)$$

Then the expression of (3.7) has a simple form when we multiply the supersymmetry parameter  $\epsilon$  to the right. Let us apply the Gamma matrices  $\Gamma^I$  to (3.7) from the right. Then the

only nonzero parts are given by

$$\begin{aligned} & [(m_1 + m_2 + m_3 - m_4 - m_5 - m_6 - m_7)^2 + (m_9 + m_{10} + m_{11} + m_{12})^2] \Gamma^3 \\ & + [(m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7)^2 + (m_9 + m_{10} + m_{11} + m_{12})^2] \Gamma^4 + \dots, \end{aligned} \quad (3.10)$$

where the abbreviated  $\Gamma^I (I = 5, 6, \dots, 10)$  terms contribute to zero when the supersymmetry parameter  $\epsilon$  is multiplied due to the constraints (3.9). Moreover, the first 12 terms of (3.8) are replaced by

$$2(m_1 + m_2 + m_3)(m_9 + m_{10} + m_{11} + m_{12})\Gamma^{3589}, \quad (3.11)$$

due to the constraint (3.9) and similarly the last 16 terms of (3.8) are written as

$$-2(m_4 + m_5 + m_6 + m_7)(m_9 + m_{10} + m_{11} + m_{12})\Gamma^{345678}. \quad (3.12)$$

Then the 33-component of  $m^2$  via  $(m^2)_{33}\Gamma^3\epsilon$  can be read off from the coefficient of  $\Gamma^3$  in (3.10) and the 44-component of  $m^2$  in  $(m^2)_{44}\Gamma^4\epsilon$  can be obtained from the coefficient of  $\Gamma^4$  in (3.10). The off-diagonal 34-component of  $m^2$  via  $(m^2)_{34}\Gamma^4\epsilon$  can be read off from the (3.11) multiplied by  $\Gamma^3$  to the right and the (3.12) multiplied by  $\Gamma^3$  to the right if one identifies the two independent nonzero components(8-th and 9-th components) of supersymmetry parameter. Finally, the off-diagonal 43-component of  $m^2$  in  $(m^2)_{43}\Gamma^3\epsilon$  can be read off from the (3.11) multiplied by  $\Gamma^4$  to the right and the (3.12) multiplied by  $\Gamma^4$  to the right. In summary, one has the following bosonic mass terms

$$\begin{aligned} (m^2)_{33} &= (m_1 + m_2 + m_3 - m_4 - m_5 - m_6 - m_7)^2 + (m_9 + m_{10} + m_{11} + m_{12})^2, \\ (m^2)_{34} &= -2(m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7)(m_9 + m_{10} + m_{11} + m_{12}), \\ (m^2)_{43} &= -2(m_1 + m_2 + m_3 - m_4 - m_5 - m_6 - m_7)(m_9 + m_{10} + m_{11} + m_{12}), \\ (m^2)_{44} &= (m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7)^2 + (m_9 + m_{10} + m_{11} + m_{12})^2. \end{aligned} \quad (3.13)$$

Therefore, for  $\mathcal{N} = 1$  supersymmetry, the two mass terms,  $(m^2)_{33}$  and  $(m^2)_{44}$ , are different due to the mass terms coming from  $\sum_{i=4}^7 m_i$  and there are crossed mass terms that will vanish if  $\sum_{i=1}^3 m_i = 0$  or  $\sum_{i=9}^{12} m_i = 0$ . Then the four-fields limit  $\sum_{i=9}^{12} m_i = 0$  leads to two independent mass terms in the deformed superpotential as in [19]. Although we have introduced 11 mass parameters in the fermionic terms in the Lagrangian, the mass terms (3.13) depend on three independent mass terms,  $\sum_{i=1}^3 m_i$  corresponding to the  $F_1^+$  four-form,  $\sum_{i=4}^7 m_i$  corresponding to the  $F_2^+$  four-form and  $\sum_{i=9}^{12} m_i$  corresponding to the  $F_3^+$  four-form. We'll come back this issue later. Thus we have found  $\mathcal{N} = 1$  superconformal Chern-Simons matter theories with global  $G_2$  symmetry. We expect that  $G_2$ -invariant  $U(N) \times U(N)$  Chern-Simons matter theory

for  $N > 2$  with level  $k = 1$  or  $k = 2$  is dual to the background of  $N$  unit of flux. We'll see other  $\mathcal{N} = 1$  theory with  $SU(3)$  global symmetry later.

- $\mathcal{N} = 2$  supersymmetry

The 1/4 BPS condition (the number of supersymmetry is 4) has the following constraints on the supersymmetry parameter  $\epsilon$  [13]

$$\Gamma^{5678}\epsilon = \Gamma^{56910}\epsilon = \Gamma^{78910}\epsilon = -\epsilon. \quad (3.14)$$

Let us further impose the conditions

$$m_1 = m_2 = m_3 = 0. \quad (3.15)$$

From the supersymmetry transformation for the bosonic fields  $\delta X_I^a = i\bar{\epsilon}\Gamma_I\Psi^a$  and the supersymmetry variation for the spinors (3.3) with (3.15), one obtains the quadratic mass terms in the Lagrangian which contains the bosonic mass terms (3.4) and the fermionic mass terms (3.2) together with (3.15). The corresponding expression of (3.5) becomes

$$\begin{aligned} \delta\mathcal{L} = & \quad ih_{ab}X_I^a(m^2)_{IJ}\bar{\Psi}^b\Gamma_J\epsilon - ih_{ab}\bar{\Psi}^a(-m_4\Gamma^{46810} + m_5\Gamma^{4679} + m_6\Gamma^{4589} + m_7\Gamma^{45710} \\ & - m_9\Gamma^{35710} - m_{10}\Gamma^{3589} - m_{11}\Gamma^{3679} + m_{12}\Gamma^{36810})^2 X_I^b\Gamma_I\epsilon. \end{aligned} \quad (3.16)$$

In order to vanish this (3.16), the relation between the bosonic and fermionic mass terms should be satisfied as before and it is given by

$$\begin{aligned} (m^2)_{IJ}\Gamma^J\epsilon = & \quad (-m_4\Gamma^{46810} + m_5\Gamma^{4679} + m_6\Gamma^{4589} + m_7\Gamma^{45710} \\ & - m_9\Gamma^{35710} - m_{10}\Gamma^{3589} - m_{11}\Gamma^{3679} + m_{12}\Gamma^{36810})^2 \Gamma_I\epsilon. \end{aligned} \quad (3.17)$$

Let us compute the quadratic mass terms in the right hand side of (3.17) in order to determine the bosonic mass terms of left hand side. The 20 terms which have the structure of Gamma matrices in (3.14) are given by

$$\sum_{i=4}^7 m_i^2 + \sum_{i=9}^{12} m_i^2 + 2(-m_4m_7 - m_5m_6 - m_9m_{12} - m_{10}m_{11})\Gamma^{5678} + \dots, \quad (3.18)$$

and the remaining 16 terms are

$$2(-m_4m_9 - m_5m_{10} - m_6m_{11} - m_7m_{12})\Gamma^{345678} + \dots. \quad (3.19)$$

Then the expression of (3.18) has a simple form when we multiply the supersymmetry parameter  $\epsilon$  to the right. Let us apply the Gamma matrices  $\Gamma^I$  to (3.18) from the right. Then the nonzero parts are given by

$$[(m_4 + m_5 + m_6 + m_7)^2 + (m_9 + m_{10} + m_{11} + m_{12})^2](\Gamma^3 + \Gamma^4) + \dots, \quad (3.20)$$

where the abbreviated  $\Gamma^I (I = 5, 6, \dots, 10)$  terms contribute to zero when the supersymmetry parameter  $\epsilon$  is added due to the constraints (3.14).

Moreover, the 16 terms of (3.19) are replaced by

$$-2(m_4 + m_5 + m_6 + m_7)(m_9 + m_{10} + m_{11} + m_{12})\Gamma^{345678}, \quad (3.21)$$

due to the constraints (3.14). The 33-component of  $m^2$  in  $(m^2)_{33}\Gamma^3\epsilon$  can be read off from the coefficient of  $\Gamma^3$  in (3.20) and the 44-component of  $m^2$  in  $(m^2)_{44}\Gamma^4\epsilon$  can be obtained from the coefficient of  $\Gamma^4$  in (3.20). The off-diagonal 34-component of  $m^2$  in  $(m^2)_{34}\Gamma^4\epsilon$  can be found from the (3.21) multiplied by  $\Gamma^3$  to the right if one identifies the four independent nonzero components (7, 8, 9-th and 10-th components in our Gamma matrices convention) of supersymmetry parameter. The off-diagonal 43-component of  $m^2$  in  $(m^2)_{43}\Gamma^3\epsilon$  can be obtained from the (3.21) multiplied by  $\Gamma^4$  to the right. Finally, one has the following bosonic mass terms

$$\begin{aligned} (m^2)_{33} &= (m_4 + m_5 + m_6 + m_7)^2 + (m_9 + m_{10} + m_{11} + m_{12})^2 = (m^2)_{44}, \\ (m^2)_{34} &= -2(m_4 + m_5 + m_6 + m_7)(m_9 + m_{10} + m_{11} + m_{12}) = -(m^2)_{43}. \end{aligned} \quad (3.22)$$

Therefore, for  $\mathcal{N} = 2$  supersymmetry, the two mass terms,  $(m^2)_{33}$  and  $(m^2)_{44}$ , are equal and there are no crossed mass terms because of  $(m^2)_{34} = -(m^2)_{43}$ . Compared with the results of four-fields in [19], as we take the zero limit of  $\sum_{i=9}^{12} m_i$ , the above mass terms become identical. The mass terms (3.22) depend on two independent mass terms,  $\sum_{i=4}^7 m_i$  corresponding to the  $F_2^+$  four-form and  $\sum_{i=9}^{12} m_i$  corresponding to the  $F_3^+$  four-form. The presence of  $\sum_{i=9}^{12} m_i$  makes the mass terms larger. Thus we have found  $\mathcal{N} = 2$  superconformal Chern-Simons matter theories with global  $SU(3) \times U(1)$  symmetry. Note that the  $\mathcal{N} = 2$  supersymmetry is encoded in  $U(1)$  factor which is R-charge. It would be interesting to find out  $SU(3) \times U(1)$ -invariant  $U(N) \times U(N)$  Chern-Simons matter theory for  $N > 2$  with level  $k = 1$  or  $k = 2$ .

Let us describe the deformed superpotential by collecting the mass terms we have found in (3.13). By redefining the masses corresponding to  $F_1^+$ ,  $F_2^+$  and  $F_3^+$  as

$$m_1 + m_2 + m_3 \equiv M_1, \quad m_4 + m_5 + m_6 + m_7 \equiv M_2, \quad m_9 + m_{10} + m_{11} + m_{12} \equiv \widehat{m}_3 \quad (3.23)$$

respectively and by introducing

$$M_2 - M_1 \equiv \widehat{m}_1, \quad M_2 + M_1 \equiv \widehat{m}_2, \quad (3.24)$$

one can write down the mass-deformed superpotential in  $\mathcal{N} = 1$  superfield notation where the fermionic independent terms are the bosonic fields in  $\Phi_7 = X_9 + \dots$  and  $\Phi_8 = X_{10} + \dots$

by reading off the mass terms (3.13) as follows:

$$\Delta W = \frac{1}{2}\hat{m}_7 \text{Tr } \Phi_7^2 + \frac{1}{2}\hat{m}_8 \text{Tr } \Phi_8^2 + \frac{1}{2}\hat{m}_{78} \text{Tr } \Phi_7 \Phi_8. \quad (3.25)$$

In the action, we should have  $\int d^3x d^2\theta W \rightarrow \int d^3x d^2\theta (W + \Delta W)$ . The  $\mathcal{N} = 1$  superpotential  $W$  before the deformation has quartic terms in  $\Phi_I^a$  and comes from the D-term and F-term of the  $\mathcal{N} = 2$  action [14, 19]. The supergravity fields  $(\rho, \varphi)$  we add in section 2 correspond to the dual of the operator  $\text{Tr } \Phi_7 \Phi_8$ . Although we have mass terms for the indices 3 and 4 in (3.13) and (3.22), we present the indices 7 and 8 instead in (3.25). As explained in [22], it depends on how one chooses the  $SU(3)$  subgroup of  $SO(8)$ . If one chooses the index of Gamma matrices differently, one can make the mass terms appearing the indices 7 and 8. Here the redefined mass terms with (3.24) are given by

$$\begin{aligned} \hat{m}_7 &\equiv (M_1 - M_2)^2 + \hat{m}_3^2 = \hat{m}_1^2 + \hat{m}_3^2, \\ \hat{m}_8 &\equiv (M_1 + M_2)^2 + \hat{m}_3^2 = \hat{m}_2^2 + \hat{m}_3^2, \\ \hat{m}_{78} &\equiv -4M_1\hat{m}_3 = 2(\hat{m}_1 - \hat{m}_2)\hat{m}_3. \end{aligned} \quad (3.26)$$

Of course, the  $\mathcal{N} = 2$  case given in (3.22) can be obtained by taking some constraints on these mass parameters.

Let us find out the phase structure characterized by the deformed superpotential (3.25) and the coefficients (3.26). When we give a mass to two of the fields ( $\hat{m}_1 = 0$  and  $\hat{m}_2 = 2\hat{m}_3$  or  $\hat{m}_2 = 0$  and  $\hat{m}_1 = 2\hat{m}_3$ ) in (3.25), then there exists a  $G_2$  symmetry. This can be easily seen since in order to get the  $G_2$ -invariant critical point and flows one should take the square of scalar field  $(\lambda, \alpha)$  in the coefficient of  $F_1^\pm$  and the square of scalar field  $(\lambda', \phi)$  in the coefficient of  $F_2^\pm$  should be equal to each other from the supergravity dual. Then one obtains  $M_1 = \pm M_2$  from (3.23). In terms of  $\hat{m}_1$  or  $\hat{m}_2$ , this implies that  $\hat{m}_1 = 0$  or  $\hat{m}_2 = 0$  from (3.24). Furthermore, the section 2 leads to the fact that the square of scalar field in the coefficient of  $F_2^\pm$  should be equal to the square of scalar field  $(\rho, \varphi)$  in the coefficient of  $F_3^\pm$  each other. Then one obtains  $\hat{m}_3 = M_2$  or  $\hat{m}_3 = M_1$  from (3.23). In terms of  $\hat{m}_1$  or  $\hat{m}_2$ , this becomes either  $\hat{m}_1 = 0$  and  $\hat{m}_2 = 2\hat{m}_3$  or  $\hat{m}_2 = 0$  and  $\hat{m}_1 = 2\hat{m}_3$  as above. We present these RG flows in the three dimensional mass parameter space in Figure 1 where the theory from  $SO(8)$ -invariant fixed point located at the origin flows a  $G_2$ -invariant fixed point when one of the masses is zero ( $\hat{m}_1 = 0$  or  $\hat{m}_2 = 0$ ) and the remaining masses satisfy either  $\hat{m}_2 = 2\hat{m}_3$  or  $\hat{m}_1 = 2\hat{m}_3$ . For the former  $G_2$  fixed point, we have  $\hat{m}_7 = \hat{m}_3^2$ ,  $\hat{m}_8 = 5\hat{m}_3^2$  and  $\hat{m}_{78} = -4\hat{m}_3^2$  from (3.26) while for the latter  $G_2$  fixed point, there are  $\hat{m}_7 = 5\hat{m}_3^2$ ,  $\hat{m}_8 = \hat{m}_3^2$  and  $\hat{m}_{78} = 4\hat{m}_3^2$ . These two fixed points are equivalent to each other.

When we give an equal mass to three fields( $\hat{m}_1 = \hat{m}_2 = \hat{m}_3$ ), then there exists  $SU(3) \times U(1)$  symmetry. How does one see this? From (3.22) and (3.26), one can easily see  $\hat{m}_1 = \hat{m}_2$  which leads to  $M_1 = 0$  and  $M_2 = \hat{m}_1 = \hat{m}_2$ . Also there is no cross term  $\hat{m}_{78} = 0$ . Furthermore, the section 2 leads to the fact that the square of scalar field( $\lambda', \phi$ ) in the coefficient of  $F_2^\pm$  should be equal to the square of scalar field( $\rho, \varphi$ ) in the coefficient of  $F_3^\pm$  each other. Then one obtains  $\hat{m}_1 = \hat{m}_2 = \hat{m}_3$  and this RG flow(along this flow the  $\mathcal{N} = 2$  supersymmetry is encoded in the  $U(1) = SO(2)$  factor which is nothing but R-charge) in the three dimensional mass parameter space is drawn in Figure 1 where the theory from  $SO(8)$ -invariant fixed point located at the origin flows the  $SU(3) \times U(1)$ -invariant fixed point when three masses are equal. For  $SU(3) \times U(1)$  fixed point, we have  $\hat{m}_7 = 2\hat{m}_3^2 = \hat{m}_8$  and  $\hat{m}_{78} = 0$  consistent with (3.22).

From the analysis of previous section, there exists a special  $\mathcal{N} = 1$  supersymmetric RG flow from the  $\mathcal{N} = 1$   $G_2$ -invariant fixed point to the  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point preserving  $SU(3)$  along the whole flow. This flow is triggered by one of the mass term( $\hat{m}_1$  or  $\hat{m}_2$ ). Starting from the  $G_2$ -invariant fixed point with  $\hat{m}_2 = 2\hat{m}_3$  and  $\hat{m}_1 = 0$ , one increases both  $\hat{m}_1$  and  $\hat{m}_3$  with fixed  $\hat{m}_2$  until  $\hat{m}_1 = \hat{m}_2 = \hat{m}_3$ . Namely, along the flow there exists  $\hat{m}_1 = 2\hat{m}_3 + \text{const}$ . Similarly starting from the  $G_2$ -invariant fixed point with  $\hat{m}_1 = 2\hat{m}_3$  and  $\hat{m}_2 = 0$ , one increases both  $\hat{m}_2$  and  $\hat{m}_3$  with fixed  $\hat{m}_1$  until  $\hat{m}_1 = \hat{m}_2 = \hat{m}_3$ . Along the flow there exists  $\hat{m}_2 = 2\hat{m}_3 + \text{const}$ . We present these RG flows in the three dimensional mass parameter space in Figure 1 where the theory from  $G_2$ -invariant fixed point flows the  $SU(3) \times U(1)$ -invariant fixed point when the masses of  $\hat{m}_1$  or  $\hat{m}_2$  are increasing.

When we give an unequal mass to three fields( $\hat{m}_1 \neq \hat{m}_2 \neq \hat{m}_3$ ), then there is a  $SU(3)$  symmetry. This is also seen from the supergravity dual analysis in the sense that for generic values of six fields there is only the least  $SU(3)$  symmetry. From the supergravity it is evident that if one has  $G_2$  flow with  $\hat{m}_1 \neq 0$  and if one turns on a small value of  $\hat{m}_2$ , then the flow is deflected to the  $SU(3) \times U(1)$ -invariant fixed point and so  $\hat{m}_2$  and  $\hat{m}_3$  grow until  $\hat{m}_1 = \hat{m}_2 = \hat{m}_3$ . Similar feature for  $G_2$  flow with  $\hat{m}_2 \neq 0$  occurs and if one turns on a small value of  $\hat{m}_1$ , the flow is deflected to the  $SU(3) \times U(1)$ -invariant fixed point and so  $\hat{m}_1$  and  $\hat{m}_3$  grow until they reach the  $SU(3) \times U(1)$ -invariant fixed point. In Figure 1, except the two  $G_2$  flow( $\hat{m}_1 = 2\hat{m}_3$  and  $\hat{m}_2 = 2\hat{m}_3$ ) and the  $SU(3) \times U(1)$  flow( $\hat{m}_1 = \hat{m}_2 = \hat{m}_3$ ) starting from the  $SO(8)$ -invariant fixed point, the generic  $\mathcal{N} = 1$  supersymmetric flows starting from the  $SO(8)$ -invariant fixed point or the  $G_2$ -invariant fixed point preserve the  $SU(3)$  symmetry.

What happen for the flows with three nonzero unequal masses( $\hat{m}_1 \neq \hat{m}_2 \neq \hat{m}_3$ )? Before the mass terms are added, the original superpotential  $W$  has terms not having  $(\Phi_7, \Phi_8)$ , terms in linear in  $\Phi_7$ , terms in linear in  $\Phi_8$  and terms that depend on  $\Phi_7$  and  $\Phi_8$ . Moreover, the deformed superpotential  $\Delta W$  is given by (3.25). When we integrate out  $(\Phi_7, \Phi_8)$  in

$W + \Delta W$  at low energy scale, we obtain the quartic terms coming from the original  $W$  which do not contain mass parameters and three kinds of sextic terms with three independent mass parameters which depend on  $\hat{m}_7, \hat{m}_8$  and  $\hat{m}_{78}$  by solving the equations of motion for  $\Phi_7$  and  $\Phi_8$  in  $W + \Delta W$ . Eventhough it is not clear in field theory that this should flow to the superconformal field theory in the IR but the gravity dual shows that it will flow to the  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point. This implies that in the IR the two independent mass parameters are equal and the other mass parameter vanishes in the resulting superpotential  $\widehat{W}$  as we take  $\hat{m}_7 = \hat{m}_8$  and  $\hat{m}_{78} = 0$  consistent with (3.22). Since the  $\mathcal{N} = 1$  theory has no R-charge, it is difficult to find out the phase structure from the field theory alone. However, the AdS/CFT provides that it can be used to study the strongly coupled field theory.

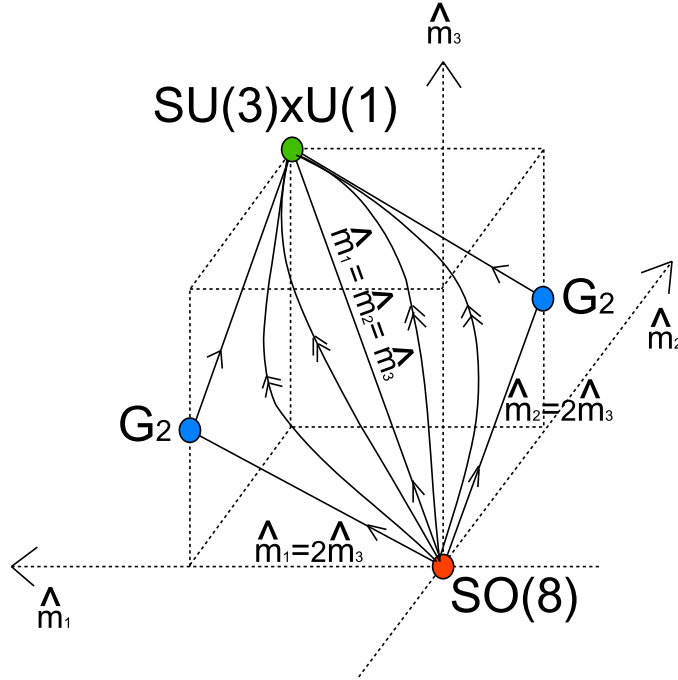


Figure 1: The RG flows starting from  $SO(8)$ -invariant fixed point in three mass parameter spaces. The theory flows a  $G_2$ -invariant fixed point when one of the masses is zero and the remaining masses are nonzero(i.e.,  $\hat{m}_1 = 2\hat{m}_3$  and  $\hat{m}_2 = 0$  or  $\hat{m}_2 = 2\hat{m}_3$  and  $\hat{m}_1 = 0$ ). If the masses are equal, then the theory flows to a  $SU(3) \times U(1)$ -invariant fixed point( $\hat{m}_1 = \hat{m}_2 = \hat{m}_3$ ). There are flows from the two  $G_2$ -invariant fixed points to  $SU(3) \times U(1)$ -invariant fixed point directly. When the masses are nonzero but not necessarily equal, the theory flows to the  $SU(3) \times U(1)$ -invariant fixed point. Except the flow for equal masses, all the flows are  $\mathcal{N} = 1$  supersymmetric in which there is a  $G_2$  symmetry along the flow ending at the  $G_2$ -invariant fixed point and otherwise there is a  $SU(3)$  symmetry.

## 4 Conclusions and outlook

A family of holographic  $\mathcal{N} = 1$  supersymmetric RG flows on M2-branes is studied. This family of flows is driven by three independent mass parameters from maximally supersymmetric  $\mathcal{N} = 8$   $SO(8)$  theory and is controlled by two IR fixed points,  $G_2$ -invariant fixed point and  $SU(3) \times U(1)$ -invariant fixed point. The generic flow with different mass parameters is  $\mathcal{N} = 1$  supersymmetric and reaches to the  $SU(3) \times U(1)$ -invariant fixed point where the three masses become identical and the supersymmetry is enhanced to  $\mathcal{N} = 2$ . There exists also a special  $\mathcal{N} = 1$  supersymmetric RG flow from the  $\mathcal{N} = 1$   $G_2$ -invariant fixed point to the  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point preserving  $SU(3)$  along the whole flow. All these flows are summarized by the Figure 1. As the title of this paper stands for, we have studied the 3-dimensional boundary Chern-Simons matter theory explicitly corresponding to the 4-dimensional bulk theory [22] where there exist six scalar fields. In [19], the phase structure of the flows with two mass parameters corresponding to turning on four scalar fields is found. The extra mass parameter in this paper, compared to [19], plays an important role. When this, corresponding to  $\hat{m}_3$  in the Figure 1, vanishes, then we reproduce the RG flows obtained in [19].

It is an open problem to uplift the four-dimensional supergravity to eleven-dimensional theory. According to [33], one can construct the eleven-dimensional metric from the solutions to four-dimensional gauged  $\mathcal{N} = 8$  supergravity. The nontrivial task is to find out the right expression for the internal four-form flux which will be present for the the most general  $SU(3)$ -singlet space of gauged  $\mathcal{N} = 8$  supergravity. So far we have considered the BPS equations (2.4) for  $\varphi = \phi, \rho = \lambda'$  in which the kinetic terms are very simple and the scalar potential can be written in terms of a superpotential. It is natural to ask whether there exist any BPS equations for general vacuum expectation values or not. Besides the two supersymmetric critical points  $\mathcal{N} = 2$   $SU(3) \times U(1)$ -invariant fixed point,  $\mathcal{N} = 1$   $G_2$ -invariant fixed point of four-dimensional gauged  $\mathcal{N} = 8$  supergravity, there are also three nontrivial nonsupersymmetric critical points as well as the trivial  $\mathcal{N} = 8$   $SO(8)$ -invariant fixed point for the scalar potential:  $SO(7)^+$ ,  $SO(7)^-$  and  $SU(4)^-$ . It is an open problem to discover any flow equations connecting any two (non)supersymmetric fixed points. For the noncompact gaugings [34, 35], it is straightforward to construct the scalar potential for the six  $SU(3)$ -singlet space.

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## References

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